Spacetime dynamics and integrable systems

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   - KdV[4], KdV/MKdV[5], Boussinesq[6].

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[3] Hawking, Perry and Strominger, 2016.

[4] Pérez, Tempo and Troncoso, 2016.

[5] Ojeda, Pérez, 2019.

[6] Ojeda, Pérez, 2020.

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   - Solitons (kink[8], breathers[9], e.g., Peregrine[10], Akhmediev[11]; Peakons[12].)
   - There is an integrable system that encompasses well known equations, e.g., KdV, MKdV, NIS and sG equations, whose name is AKNS system[13].

---

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- Impose boundary conditions to the gravitational field.
- Study the consistency of the boundary conditions.
- Recover an associated metric from the boundary dynamics.

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where $r = r(t, \phi)$ and $p = p(t, \phi)$ are dynamical fields, $A(t, \phi)$, $B(t, \phi)$ and $C(t, \phi)$ are functions that has to be specified and ξ is a constant.

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$$A = \sum_{n=0}^{N} A_n \xi^{N-n}, \;\; B = \sum_{n=0}^{N} B_n \xi^{N-n}, \;\; C = \sum_{n=0}^{N} C_n \xi^{N-n}.$$

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\begin{aligned}
A_n' &= pC_n - rB_n, \\
B_{n+1} &= -\frac{1}{2}B_n' - pA_n, \\
C_{n+1} &= \frac{1}{2}C_n' - rA_n, \\
B_0 &= C_0 = 0,
\end{aligned}
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Equating order by order in ξ, we obtain a set of equations from the coefficients associated to the $n-$th power of the spectral parameter: That is, a set of recurrence relations,

$$A'_n = pC_n - rB_n,$$
$$B_{n+1} = -\frac{1}{2}B'_n - pA_n,$$
$$C_{n+1} = \frac{1}{2}C'_n - rA_n,$$
$$B_0 = C_0 = 0,$$

with dynamic equations

$$\boxed{\dot{p} = -B'_N - 2pA_N},$$
$$\boxed{\dot{r} = -C'_N + 2rA_N}.$$

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The AKNS system

According to the obtained recurrence relations, it is possible to construct the first terms A_n, B_n and C_n,

$$A_0 = 1, \quad A_1 = 0, \quad A_2 = -\frac{1}{2}pr, \quad A_3 = \frac{1}{4}\left(p'r - pr'\right),$$

$$B_0 = 0, \quad B_1 = -p, \quad B_2 = \frac{1}{2}p', \quad B_3 = \frac{1}{2}p^2 r - \frac{1}{4}p'',$$

$$C_0 = 0, \quad C_1 = -r, \quad C_2 = -\frac{1}{2}r', \quad C_3 = \frac{1}{2}pr^2 - \frac{1}{4}r''.$$

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For $N = 3$,

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where, if $r = -1$, we obtain, in particular, the KdV equation,

$$\dot{p} = \frac{3}{2}pp' + \frac{1}{4}p''',$$

while, for $p = -r$, the MKdV equation

$$\dot{p} = \frac{3}{2}p^2p' + \frac{1}{4}p'''.$$

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The AKNS system

For $N = 2$,

$$\dot{p} = p^2 r - \frac{1}{2}p'', \quad \dot{r} = -pr^2 + \frac{1}{2}r'',$$

we get the (Wicked rotated) nonlinear Schrödinger equation

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we get the (Wicked rotated) nonlinear Schrödinger equation

- The Sine-Gordon equation is also included in this framework, however, negative powers of ξ must be included in the expansion in order to make it apparent.

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The AKNS system

Following recursive methods, AKNS realized that the system has infinite conserved charges,

$$H_2 = -\int d\phi \, pr, \quad H_3 = \frac{1}{4}\int d\phi \, \left(p'r - pr'\right), \quad \ldots$$

The AKNS system

The AKNS system may be written as a bi-Hamiltonian system[14]

$$\begin{pmatrix} \dot{r} \\ \dot{p} \end{pmatrix} = \mathcal{D}_1 \begin{pmatrix} \mathcal{R}_{N+1} \\ \mathcal{P}_{N+1} \end{pmatrix} = \mathcal{D}_2 \begin{pmatrix} \mathcal{R}_{N+2} \\ \mathcal{P}_{N+2} \end{pmatrix},$$

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where

$$\mathcal{D}_1 = \begin{pmatrix} -2r\partial_\phi^{-1}(r\cdot\ ) & -\partial_\phi + 2r\partial_\phi^{-1}(p\cdot\ ) \\ -\partial_\phi + 2p\partial_\phi^{-1}(r\cdot\ ) & -2p\partial_\phi^{-1}(p\cdot\ ) \end{pmatrix}, \quad \mathcal{D}_2 = \begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix},$$

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with

$$A_n = \frac{n-1}{2}\mathcal{H}_n, \quad B_n = \mathcal{R}_{n+1}, \quad C_n = \mathcal{P}_{n+1},$$

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Furthermore, the conserved charges are related by the following recursion formula

$$\mathcal{D}_2 \begin{pmatrix} \mathcal{R}_{n+1} \\ \mathcal{P}_{n+1} \end{pmatrix} = \mathcal{D}_1 \begin{pmatrix} \mathcal{R}_n \\ \mathcal{P}_n \end{pmatrix}, \quad n = 1, 2, 3, \ldots$$

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AdS_3 general relativity

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$$I = I_{CS}[\mathcal{A}^+] - I_{CS}[\mathcal{A}^-],$$

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$$L_{-1}^\pm = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\ ,\ L_0^\pm = \begin{pmatrix} -1/2 & 0 \\ 0 & 1/2 \end{pmatrix},\ L_1^\pm = \begin{pmatrix} 0 & -1 \\ 0 & 0 \end{pmatrix},$$

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where the $sl(2,\mathbb{R})$ algebra is

$$\left[L_n^{\pm}, L_m^{\pm}\right] = (n-m)L_{n+m}. \quad (1)$$

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AdS_3 general relativity

The connection splits as $\mathcal{A} = \mathcal{A}^+ + \mathcal{A}^-$, where

$$\mathcal{A}^\pm = \omega \pm \frac{e}{l},$$

with $e^a = e^a_\mu dx^\mu$ the vielbein and $\omega^a_\mu dx^\mu$ the spin connection.

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with $e^a = e^a_\mu dx^\mu$ the vielbein and $\omega^a_\mu dx^\mu$ the spin connection. The action is

$$I_{CS}[\mathcal{A}^\pm] = \frac{k}{4\pi}\int \left\langle \mathcal{A}^\pm d\mathcal{A}^\pm + \frac{2}{3}\mathcal{A}^{\pm 3} \right\rangle,$$

and $F^\pm = d\mathcal{A}^\pm + \mathcal{A}^{\pm 2} = 0$.

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Hamiltonian formalism

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where $F_{ij}^\pm = \partial_i \mathcal{A}_j^\pm - \partial_j \mathcal{A}_i^\pm + \left[\mathcal{A}_i^\pm, \mathcal{A}_j^\pm\right]$.

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where $F_{ij}^\pm = \partial_i \mathcal{A}_j^\pm - \partial_j \mathcal{A}_i^\pm + \left[\mathcal{A}_i^\pm, \mathcal{A}_j^\pm\right]$. We can see that $\mathcal{A}_t^\pm$ is a Lagrange multiplier and $F_{ij}^\pm$ a constraint of the theory.

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Boundary term

If we want a bonna fide action principle, we must suplement the action with a surface integral, which is

$$\delta\mathcal{B}^{\pm} = -\frac{k}{2\pi}\oint_{\rho\to\infty} dt d\phi \left\langle A_t^{\pm}\,\delta A_\phi^{\pm}\right\rangle.$$

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$$\delta\mathcal{B}^{\pm} = -\frac{k}{2\pi}\oint_{\rho\to\infty} dt d\phi \left\langle A_t^{\pm}\,\delta A_\phi^{\pm}\right\rangle.$$

Hence, we must specify $A_t^{\pm}(t,\rho\to\infty,\phi)$ and $A_\phi^{\pm}(t,\rho\to\infty,\phi)$, in order to integrate the surface term.

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Before proceeding, it is convenient to gauge-away the radial dependence. When $\mathcal{A}^{\pm}$ transforms as,

$$a^{\pm} = b_{\pm}^{-1}\left(d + \mathcal{A}^{\pm}\right) b_{\pm},$$

the choice $b_{\pm}(\rho) = e^{\pm\rho L_0^{\pm}}$ captures completely the radial dependence, yielding

$$a_{\rho}^{\pm} = 0, \quad a_t^{\pm} = a_t^{\pm}(t,\phi), \quad a_{\phi}^{\pm} = a_{\phi}^{\pm}(t,\phi).$$

Boundary conditions

The boundary conditions are

$$a_\phi^\pm = \mp 2\xi^\pm L_0 - p^\pm L_{\pm 1} + r^\pm L_{\mp 1},$$
$$a_t^\pm = \frac{1}{\ell}(-2A^\pm L_0 \pm B^\pm L_{\pm 1} \mp C^\pm L_{\mp 1}),$$

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where $p^\pm = p^\pm(t,\phi)$ and $r^\pm = r^\pm(t,\phi)$ are the fields carrying the boundary dynamics of the theory, $A^\pm = A^\pm(t,\phi)$, $B^\pm = B^\pm(t,\phi)$ and $C^\pm = C^\pm(t,\phi)$ are polynomials functions on $\xi^\pm$ that has to be specified.

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Boundary conditions

Thus, the zero-curvature equation of motion

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$$f_{t\phi}^{\pm} = \partial_t a_\phi^{\pm} - \partial_\phi a_t^{\pm} + \left[a_t^{\pm}, a_\phi^{\pm}\right] = 0,$$

yields the AKNS system (but in the Chern-Simons formulation)

$$\begin{aligned} \pm\dot{r}^{\pm} + \frac{1}{\ell}\left(C'^{\pm} - 2r^{\pm}A^{\pm} - 2\xi^{\pm}C^{\pm}\right) &= 0, \\ \pm\dot{p}^{\pm} + \frac{1}{\ell}\left(B'^{\pm} + 2p^{\pm}A^{\pm} + 2\xi^{\pm}B^{\pm}\right) &= 0, \\ A'^{\pm} - p^{\pm}C^{\pm} + r^{\pm}B^{\pm} &= 0. \end{aligned}$$

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A remark.

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As a result, the functions α, β and γ are

$$\alpha = \sum_{m=0}^{M} \frac{(m-1)}{2} \mathcal{H}_m \xi^{M-m}, \quad \beta = \sum_{m=0}^{M} \mathcal{R}_{m+1} \xi^{M-m}, \quad \gamma = \sum_{m=0}^{M} \mathcal{P}_{m+1} \xi^{M-m},$$

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where M is a positive integer and labels an infinite family of permissible gauge transformations. Hence, the infinitesimal transformation of the fields r and p are

$$\delta r = -\gamma'_M + 2r\alpha_M, \quad \delta p = -\beta'_M - 2p\alpha_M.$$

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It is possible to prove that the algebra of charges is

$$\{Q[\Lambda], Q[\overline{\Lambda}]\} = 0.$$

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where $p^{\pm} = \sum_n p_n^{\pm} e^{in\phi}$ and $r^{\pm} = \sum_n r_n^{\pm} e^{in\phi}$.

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Remarkably, the three above configurations are attainable.

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It is important to say that any solution of Einstein's equations in three dimensions with a negative cosmological constant corresponds to a spacetime of constant negative curvature.

Thus, its geometry coincides locally AdS_3. The only difference resides in its global properties[18].

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Construction of the metric

We can recover the metric,

$$g_{\mu\nu} = \frac{\ell^2}{2} \left\langle \left(A_\mu^+ - A_\mu^- \right) \left(A_\nu^+ - A_\nu^- \right) \right\rangle .$$

In ADM coordinates

$$ds^2 = -N^2 dt^2 + \gamma_{ij}(N^i dt + dx^i)(N^j dt + dx^j),$$

with $i = \rho, \phi$, we obtain

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Construction of the metric

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the spatial metric

$$\gamma_{ij} = \begin{pmatrix} \frac{\ell^2}{\rho^2} & -\frac{\ell^2}{\rho}\left(\xi^+ + \xi^-\right) \\ -\frac{\ell^2}{\rho}\left(\xi^+ + \xi^-\right) & \ell^2\left(\xi^+ + \xi^-\right)^2 + \rho^2 \omega^- \omega^+ \end{pmatrix},$$

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Construction of the metric

where the auxiliary functions $\Omega^{\pm}$ and $\omega^{\pm}$ are defined as

$$\Omega^{\pm} \equiv B^{\pm} - \frac{\ell^2}{\rho^2} C^{\mp}, \quad \omega^{\pm} \equiv p^{\pm} + \frac{\ell^2}{\rho^2} r^{\mp},$$

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Final remarks

The familiy of boundary conditions constructed here encompasses some examples found in the literature.

- The Brown-Henneaux boundary conditions[20] may be recovered when $N^{\pm} = 1$, $r^{\pm} = 1$ and then setting $\xi^{\pm} = 0$.

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2. We studied its spacetime geometrization in $2+1$ dimensions, i.e., we constructed a bonna fide action principle, we found its asymptotic symmetries, we computed the algebra of charges and we proved that gravitational configurations, such as black holes, are attainable.
3. Further work must be done, such as thermodynamics, higher spins extensions, Hamiltonian reduction, etc.

Thank you for your attention!